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Nuclear Modification to Parton Distribution Functions and Parton Saturation

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Abstract. We introduce a generalized definition of parton distribution functions (PDFs) for a more consistent all-order treatment of power corrections. We present a new set of modified DGLAP evolution equations for nuclear PDFs, and show that the resummed $\alpha_s A^{1/3}/Q^2$-type of leading nuclear size enhanced power corrections significantly slow down the growth of gluon density at small-x. We discuss the relation between the calculated power corrections and the saturation phenomena.

1. Introduction

RHIC has produced good evidence that a new state of hot and dense matter of quarks and gluons, the quark-gluon plasma (QGP), was formed in ultra-relativistic heavy ion collisions [1]. To extract useful information on QGP properties, we need to understand the dynamics of parton multiple scattering and to derive precise nuclear PDFs (nPDFs) to calibrate the production of hard probes. In this talk, we use lepton-hadron deep inelastic scattering (DIS) as an example to demonstrate a need to modify the definition of PDFs for all-order treatments of parton multiple scattering and power corrections [2]. We present a new set of parton evolution equations consistent to QCD factorization beyond leading power and show that one-loop $\alpha_s A^{1/3}/Q^2$-type of power corrections significantly slow down the growth of gluon density at small-x. We argue that power corrections are very important for understanding the transition from a region where leading power pQCD has been very successful to the regime of parton saturation where new and novel strong interaction phenomena emerge [3].

2. QCD factorization and parton distribution functions

Under the one-photon approximation, DIS cross section is determined by the hadronic tensor, $W^{\mu\nu}(P,q)$, of hadron momentum P and virtual photon momentum q, represented by the imaginary part of partonic scattering diagrams in Fig. 1. When $Q^2 = -q^2$ is much greater than the hadron size 1/fm, diagrams in Fig. 1 are dominated by the phase space near $k^2 \sim (1/\text{fm})^2 \ll Q^2$ due to the perturbative pinch singularity at $k^2 \sim 0$ [4].

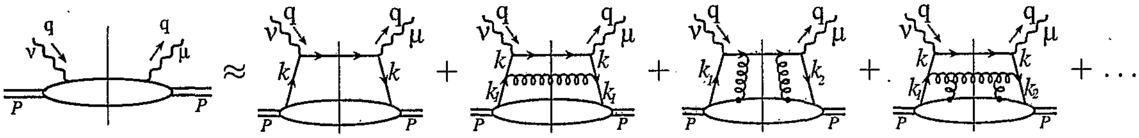

Figure 1. Feynman diagram expansion of DIS hadronic tensor $W_{\mu\nu}$.

By applying the operator product expansion, the hadronic tensor can be systematically expanded in a power series of $1/Q^2$ and perturbatively factorized as [5, 6],

$$W^{\mu\nu}(x_B, Q^2) = \sum_{n=0} \left(\frac{1}{Q^2}\right)^n \mathcal{H}^{\mu\nu}_{2+2n}\left(\frac{x_B}{\{x_i\}}, \frac{Q^2}{\mu^2}\right) \otimes f_{2+2n}(\{x_i\}, \mu^2) \tag{1}$$

where $x_B = Q^2/2P\cdot q$, $\otimes$ represents the convolution of parton momentum fractions x_i with $i = 1+2n$, f_2's are PDFs, f_i's with $i \geq 4$ are twist-i parton correlation functions (PCFs), and $\mathcal{H}^{\mu\nu}_i$ are corresponding photon-parton scattering amplitudes with all perturbative collinear divergences removed. In Eq. (1) and below, we suppress parton flavor dependence. The $\mathcal{H}^{\mu\nu}_i$ can be perturbatively computed order-by-order in powers of α_s by applying Eq. (1) to a partonic state. At leading power, tree-level quark scattering amplitude provides parton model formalism. Radiative corrections at $\mathcal{O}(\alpha_s)$ can be derived, for example, in Fig. 2, by calculating the $\mathcal{O}(\alpha_s)$ quark scattering amplitude on the left and $\mathcal{O}(\alpha_s)$ quark distribution on the right. QCD factorization assures that

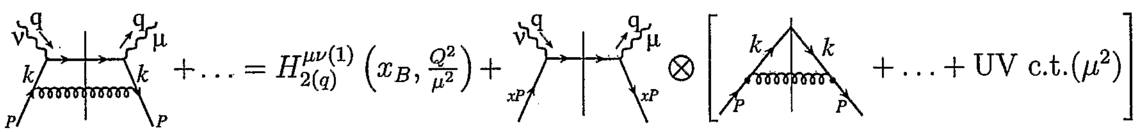

Figure 2. Sample calculation of leading power partonic tensor at NLO.

perturbative collinear divergence of the quark scattering amplitude on the left is exactly canceled by the same divergence of quark distribution on the right, and $\mathcal{H}^{\mu\nu}_{2(q)}$ is infrared safe. The virtuality of the loop momentum k of the scattering amplitude on the left is limited by the kinematics at $\mathcal{O}(Q^2)$. The k in quark distribution on the right leads to a UV divergence that is removed by imposing a counterterm at a factorization scale $\mu^2 \sim \mathcal{O}(Q^2)$. From Fig. 2, all low mass states of the quark scattering amplitude at $|k^2| \leq \mu^2$ are canceled by that of the quark distribution on the right, and $\mathcal{H}^{\mu\nu}_{2(q)}$ keeps only the short-distance scattering dynamics at $\mathcal{O}(Q^2)$. The μ^2 dependence of $\mathcal{H}^{\mu\nu}_{2(q)}$ is an immediate consequence of the definition of parton-level quark distribution in Fig. 2, and leads to DGLAP evolution equations of the PDFs.

3. Power corrections and modified parton evolution

To expand the reach of pQCD and explore the transition region to nonperturbative physics, systematic and reliable calculations of power corrections in Eq. (1) are crucial. When $x_B \ll x_c = 1/2mR \sim 0.1$ with hadron mass m and radius R, the virtual photon in DIS could cover whole hadron in longitudinal direction to enhance power corrections [4]. Power corrections from tree-level diagrams were recently computed and found to be important at small x_B [7]. The $\mathcal{O}(\alpha_s)$ correction could be computed in a way similar to the leading power. For example, in Fig. 3, $\mathcal{H}^{\mu\nu(2)}_{4(qg)}$ is infrared safe with all collinear divergence canceled between the left and the right. However, unlike the leading power

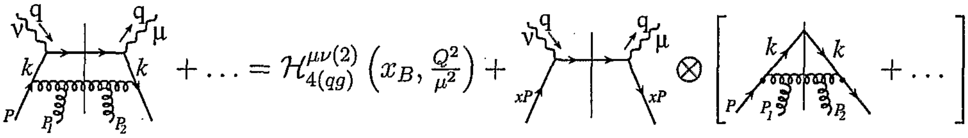

Figure 3. Sample calculation of $1/Q^2$ power corrections at NLO.

case in Fig. 2, the k-integration of the diagram for the quark distribution on the right is UV finite, and consequently, $\mathcal{H}^{\mu\nu(2)}_{4(qg)}$ contains the physics of high mass states ($|k^2| > Q^2$) that were not in the partonic scattering amplitude on the left.

Physically, we want *all* short-distance coefficient functions in QCD factorization formulas, like $\mathcal{H}^{\mu\nu}_i$ in Eq. (1), to contain only dynamics of partonic scattering at $\mathcal{O}(Q^2)$. We can achieve this by defining the PDFs in Fig. 3 to contain only mass states with $|k^2| < \mu^2 \sim Q^2$. Consequently, from this additional μ^2 dependence of PDFs, DGLAP parton evolution equations receive power corrections [2, 8]

$$\mu^2 \frac{\partial}{\partial \mu^2} f_2(x, \mu^2) = \sum_{n=0} \left(\frac{1}{\mu^2}\right)^n \mathcal{P}_{2+2n}\left(\frac{x}{\{y_i\}}\right) \otimes f_{2+2n}(\{y_i\}, \mu^2)\,, \tag{2}$$

where f_i with $i \geq 4$ obey similar equations to form a closed set of evolution equations [2]. The $\mathcal{P}$'s can be perturbatively computed from μ^2 dependence of PDFs and PCFs.

If the parton momentum $x \gg x_s$ with saturation scale $Q_s^2(x_s) = \mu^2$ [3], the one-loop contribution to quark distribution in Fig. 3 is dominated by diagrams with two un-pinched poles that fix two of the three parton momentum integrals. Under this leading pole approximation [9], only diagrams with maximum un-pinched poles contribute and all y_i but one in Eq. (2) are fixed by these poles. We computed all one-loop leading $\alpha_s A^{1/3}/Q^2$-type of power corrections to parton evolution of nPDFs [2]

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y}$$
$$\times \begin{pmatrix} P_{qq}(\frac{x}{y})\hat{T}_{\Delta'} & P_{qg}(\frac{x}{y})\hat{T}_{\Delta} \\ \sum_{i=1}^{2n_f} P_{gq}(\frac{x}{y})\hat{T}_{\Delta} & P_{gg}(\frac{x}{y})\hat{T}_{\Delta'} - \frac{n_f}{3}\delta(1-\frac{x}{y})\hat{T}_{\Delta} \end{pmatrix} \begin{pmatrix} q(y,\mu^2) \\ g(y,\mu^2) \end{pmatrix} \tag{3}$$

where $\hat{T}_{\Delta}$ is a translation operator that shifts momentum fraction $y \to y(1+\Delta)$ with $\Delta = \frac{C_F}{C_A}\Delta' = \xi^2(A^{1/3}-1)/Q^2$ and ξ^2 defined in [7], and P_{qq}, P_{qg}, P_{gq} are normal leading order splitting functions, while P_{gg} is the normal $g \to g$ splitting function with the term proportional to n_f excluded. In Fig. 4(a) and (b), we show gluon evolution without (red) and with (blue) power corrections, and the ratio of these two different evolutions, respectively. Using three sets of available PDFs, we found that the resummed power corrections can give as large as 20% reduction in *slope* of gluon evolution and can significantly slow down the growth of gluon density at small-x and low Q^2.

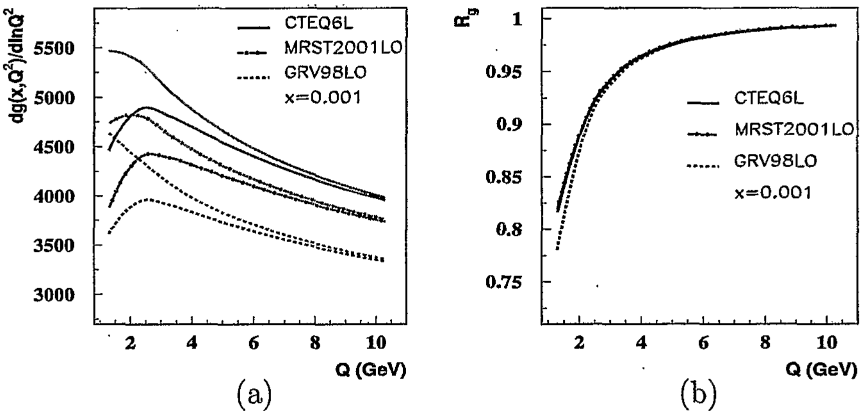

Figure 4. Modification of gluon evolution for nucleus of $A = 197$

4. Summary and outlook

We modified the definition of PDFs so that all short-distance coefficient functions of power corrections in QCD factorization formulas contain only dynamics of partonic scattering at $\mathcal{O}(Q^2)$. Consequently, we obtained a new set of parton evolution equations including calculable power corrections in Eq. (2). Under the leading pole approximation, we computed all one-loop contributions to the evolution equations of nPDFs and found that leading $\alpha_s A^{1/3}/Q^2$-type power corrections significantly slow down the growth of gluon density at small-x and drive the gluon density to saturation [2].

Our factorization treatment of power corrections can be applied to any factorisable observables. Power corrections closely connect to coherent multi-parton dynamics and are important for studying the transition from a dilute to a saturated partonic system. When partons are close to be saturated, or all x_i are close to the x_s, the leading pole approximation is no longer reliable. Many more diagrams will contribute. We need different approaches for studying power corrections and parton correlations [3, 8].

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References

[1] I. Arsene *et al.* [BRAHMS Collaboration], Nucl. Phys. A **757**, 1 (2005); B. B. Back *et al.*, Nucl. Phys. A **757**, 28 (2005); J. Adams *et al.* [STAR Collaboration], Nucl. Phys. A **757**, 102 (2005); K. Adcox *et al.* [PHENIX Collaboration], Nucl. Phys. A **757**, 184 (2005).
[2] Z. B. Kang and J. W. Qiu, in preparation.
[3] A.H. Mueller, Nucl. Phys. B **558**, 285 (1999);
E. Iancu and R. Venugopalan, arXiv:hep-ph/0303204; and references therein.
[4] J. W. Qiu, Nucl. Phys. A **715**, 309 (2003), and references therein.
[5] G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B **175** (1980) 27.
[6] J. W. Qiu and G. Sterman, Nucl. Phys. B **353**, 105 (1991); B **353**, 137 (1991).
[7] J. W. Qiu and I. Vitev, Phys. Rev. Lett **93**, 262301 (2004); and references therein.
[8] A. H. Mueller and J. W. Qiu, Nucl. Phys. B **268**, 427 (1986).
[9] J. W. Qiu and G. Sterman, Int. J. Mod. Phys. E **12**, 149 (2003), and references therein.